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# Comment on "Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces with radiation" [E.M.E. Elbarbary, N.S. Elgazery, International Journal of Thermal Sciences 43 (2004) 889–899]

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In the above paper an analysis has been carried out to obtain results in a steady laminar boundary layer flow over a vertical plate moving parallel or reversely to a vertical free stream in a micropolar fluid in a Darcy porous medium. The plate temperature is constant and different from the free stream temperature while radiation has been taken into account in the energy equation. The fluid viscosity and thermal conductivity are assumed to be functions of temperature. The boundary layer equations are transformed into ordinary ones and subsequently are solved using the Chebyshev finite difference method. However, there are some errors in the above paper which are presented below:

In page 890 it is mentioned that the fluid is electrically conducting. However, this property is related to an applied magnetic field and in the above work no magnetic field exists.

In page 890 values are given for some constants which concern the variation of viscosity and thermal conductivity with temperature. However, these constants concern water, air and lubrication oils whereas the above problem concern micropolar fluids. There is no relation between water, air and lubrication oils with micropolar fluids.

In page 892 it is mentioned that "The present work deals with application of a radically new approach to computation of the boundary layer equations in MHD flows". However, no magnetic field exists in the above work.

The momentum equation used by the authors (Eq. (2) in their paper) is

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{\rho}\frac{\partial}{\partial y}\left[(\mu + K)\frac{\partial u}{\partial y}\right] + \frac{K}{\rho}\frac{\partial N}{\partial y} - \frac{\nu}{k^*}u \tag{1}$$

where $u$ and $v$ are the velocity components, $\mu$ is the fluid dynamic viscosity, $\rho$ is the fluid density, $K$ is the vortex viscosity, $N$ is the angular velocity, $\nu$ is the fluid kinematic viscosity and $k^*$ is the permeability of the porous medium. The boundary conditions are (Eq. (5) in their work):

$$\text{at } y = 0: \quad u = u_w,\ v = 0,\ N = 0,\ T = T_w \tag{2}$$

$$\text{as } y \to \infty \quad u = u_\infty,\ N = 0,\ T = T_\infty \tag{3}$$

Let us apply the momentum equation at large $y$. At large distances from the plate ($y \to \infty$) the fluid velocity is everywhere constant and equal to $u_\infty$ while the velocity gradient $\partial u/\partial y$ is also zero. The same happens with the angular velocity gradient $\partial N/\partial y$. This means that the momentum equation takes the following form at large $y$

$$u_\infty\frac{\partial u_\infty}{\partial x} = -\frac{\nu}{k^*}u_\infty \tag{4}$$

or

$$\frac{\partial u_\infty}{\partial x} = -\frac{\nu}{k^*} \tag{5}$$

From the above equation we see that the free stream velocity should change along $x$ and therefore the momentum equation (1) is not compatible with the assumption that the free stream velocity is constant. Taking into account the above finding it is clear that the momentum equation is wrong.

It is known in boundary layer theory that velocity and temperature profiles approach the ambient fluid conditions asymptotically and do not intersect the line which represents the boundary conditions. This demand exist also in the above work taking into account the boundary conditions (10) in their paper. In Fig. 1 we show a temperature profile taken from Fig. 9(a) of the above work. We see that the temperature profile does not approach the ambient condition asymptotically but intersects the

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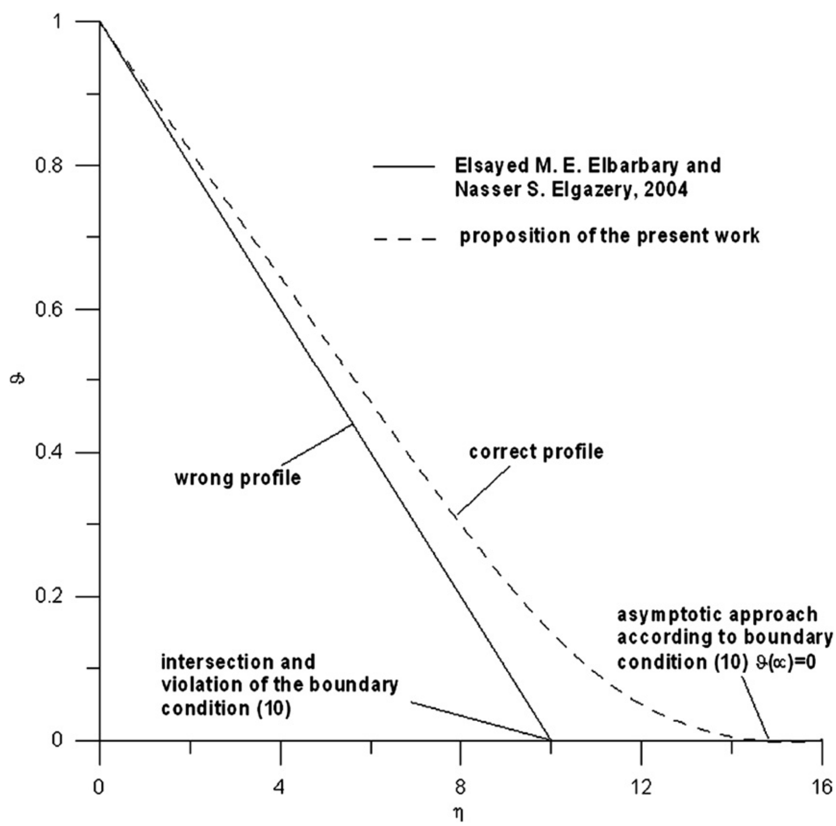


Fig. 1. The solid line represents a dimensionless temperature profile for $\gamma_1 = 1$, $\vartheta_r = 0.01$, $\Delta = 0.5$, $S = 0.5$, $F = 1$, $r = 0.3$ and $k_p = 0.05$. This profile has been reproduced from Fig. 9(a) by E.M.E. Elbarbary and N.S. Elgazery [1].

horizontal axis with a steep angle (the temperature profile is a straight line). At the same figure we show the correct shape of this temperature profile. It is clear that this temperature profile given by E.M.E. Elbarbary and N.S. Elgazery [1] is wrong and this happens in many figures. All temperature profiles included in Fig. 2, all velocity profiles in Fig. 3(b), many velocity profiles in Fig. 5, all temperature profiles in Fig. 6(a), all temperature profiles in Fig. 6(b), one velocity profile in Fig. 7(b), all temperature profiles in Fig. 9(a), all temperature profiles in Fig. 9(b), one temperature profile in Fig. 10(a), all temperature profiles in Fig. 10(b), all temperature profiles in Fig. 11, three temperature profiles in Fig. 14(a) and all temperature profiles in Fig. 14(b) intersect the horizontal axis and are wrong. It is clear that the profiles which do not approach the horizontal axis asymptotically and intersect it, are truncated due to a small calculation domain used. Probably the calculation domain was not sufficient to capture the real shape of profiles and a wider calculation domain should be used. It is sure that the truncation of the profiles has introduced errors and in the numerical results included in Tables 1(a), 1(b), 2(a), 2(b) of the above paper.

## References

[1] E.M.E. Elbarbary, N.S. Elgazery, Chebyshev finite difference method for the effects of variable viscosity and variable thermal conductivity on heat transfer from moving surfaces with radiation, International Journal of Thermal Sciences 43 (2004) 889–899.